

# Shakedown modeling of unsaturated expansive soils subjected to wetting and drying cycles

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**Abstract.** It is important to model the behavior of unsaturated expansive soils subjected to wetting and drying cycles because they alter significantly their hydro-mechanical behavior and therefore cause a huge differential settlement on shallow foundations of the structure. A simplified model based on the shakedown theory (Zarka method) has been developed in this study for unsaturated expansive soils subjected to wetting and drying cycles. This method determines directly the stabilized limit state and consequently saves the calculation time. The parameters of the proposed shakedown-based model are calibrated by the suction-controlled oedometer tests obtained for an expansive soil compacted at loose and dense initial states, and then validated for the same soil compacted at intermediate initial state by comparing the model predictions with the experimental results. Finally, the finite element equations for the proposed shakedown model are developed and these equations are implemented in the finite element code CAST3M to carry out the full-scale calculations. A 2D geometry made up of the expansive soil compacted at the intermediate state is subjected to successive extremely dry and wet seasons for the different applied vertical loads. The results show the swelling plastic deformations for the lower vertical stresses and the shrinkage deformations for the higher vertical stresses.

## 1 Introduction

Unsaturated expansive soils contain clay minerals such as smectite with the high capacity of water absorption. As the clay minerals absorb water, they expand; conversely, as they lose water, they shrink. The successive wetting and drying cycles produce the differential settlements and crack the structures built on this material [1-6]. To investigate the hydro-mechanical behavior of unsaturated expansive soils, it is necessary to determine the coupled behavior of expansive soils for the different applied loads and environmental solicitations.

Nowamooz and Masrouri [7], have performed the tests on loosely and densely compacted bentonite/silt mixtures for several wetting and drying cycles. In their work, the authors pointed out that the loose samples present a shrinkage strain accumulation while a swelling strain accumulation can be observed for dense samples before reaching a unique equilibrium state at the end of the wetting and drying cycles, regardless of the initial dry density of these expansive soil samples. In other words, the loose and dense samples will achieve a unique final state after a number of wetting and drying cycles.

Many methods for volume change prediction of unsaturated expansive soils have been proposed [8-12]. Barcelona Expansive Model (BExM) developed by Alonso et al. [12], can be mentioned as the most accepted theoretical reference. This model is able to simulate the basic behavior of unsaturated expansive soil, including

the strain fatigue phenomenon during wetting-drying cycles and the prediction of final equilibrium state at the end of the wetting-drying cycles. However, this model presents a large number of parameters such as the coupling functions for micro- and macrostructural strains. The calibration of these parameters needs several experimental tests which lead to a time-consuming procedure to characterize their hydro-mechanical behaviors.

The shakedown theory has been simplified by Zarka [13,14] for kinematic hardening materials such as metals. Several authors [15-19] then applied this method to study the cyclic mechanical behavior of soils. They introduced a series of transformed internal variables to characterize the mechanical system and then constructed a local geometry in transformed internal parameter plane to estimate the stabilized limit state and its associated plastic components. This direct determination of the steady solutions in shakedown analysis is able to replace classic step-by-step method and needs less model parameters.

In this context, this paper presents a shakedown-based model for the simulation of the volume change problem in expansive soils subjected to several wetting and drying cycles. The constitutive model based on shakedown theory (Zarka method) is firstly presented. The required parameters of this model are next calibrated and validated by the experimental results obtained for an expansive soil compacted at loose, intermediate and dense initial states. After the implementation of the shakedown theory in the

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finite element code, the plastic strain field and the inelastic displacement field are finally calculated for a 2D geometry consisted of the intermediately compacted expansive soil and subjected to successive wetting and drying cycles.

## 2 Analytical modeling of expansive soils based on shakedown concept

The purpose of this section is to present a simplified shakedown-based model to simulate the hydro-mechanical behaviour of unsaturated expansive soils during the successive wetting and drying cycles.

It is generally accepted that a unique plane (net mean stress-suction) is sufficient to describe the hydro-mechanical behaviour of unsaturated soils. Figure 1 shows in this plane a rectangular yield surface representing the elastic domain. The equations of the different boundaries can be given by:

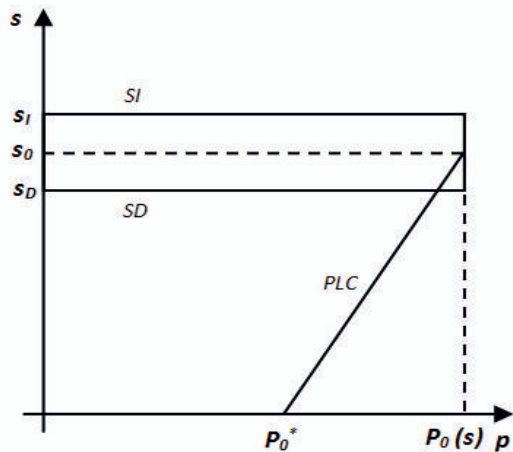
$$s = s_I \quad (1)$$

$$s = s_D \quad (2)$$

$$p = p_0 \quad (3)$$

where,  $s_I$  is the Suction Increase limit,  $s_D$  is the Suction Decrease limit and  $p_0$  is the Preconsolidation Stress.

In this work, we take into account the width of the elastic domain is small and is not changing with suction cycles.



**Figure 1.** Rectangular shape for the yield surfaces in (suction - preconsolidation stress) plane.

The suction variation within the rectangular will result in the elastic volumetric strain:

$$d\epsilon_{vs}^e = \frac{\kappa_s}{v} \cdot \frac{ds_0}{s_0} \quad (4)$$

and if the boundaries ( $s = s_I$  and  $s = s_D$ ) are activated, the following total and plastic volumetric strain will be generated,

$$d\epsilon_{vs} = \frac{\lambda_s}{v} \cdot \frac{ds_0}{s_0} \quad (5)$$

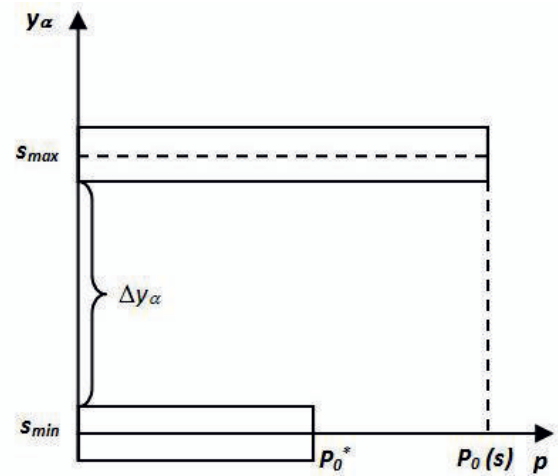
$$d\epsilon_{vs}^p = \frac{\lambda_s - \kappa_s}{v} \cdot \frac{ds_0}{s_0} \quad (6)$$

in which,  $\kappa_s$  and  $\lambda_s$  are elastic stiffness index and elastoplastic stiffness index for suction variation, respectively.

### 2.1 Plastic Shakedown during suction cycles

When the loading amplitude becomes very large, a stabilized limit state can be reached at the end of the suction cycles. The transformed internal parameter ( $y_\alpha$ ) as well as the preconsolidation stress ( $p$ ) is presented in the same plane in Figure 2. In the (transformed internal parameter - preconsolidation stress) plane, the convex which characterizes the behaviour of the soil sample translates between the minimum suction ( $s_{min}$ ) position and the maximum suction ( $s_{max}$ ) position during the wetting and drying cycles. If the extreme positions of the convex have no common part on the transformed internal parameter axis, plastic shakedown will occur. Therefore, the variation of the volumetric plastic deformation ( $\Delta\epsilon_{vs}^p$ ) during suction cycles can be computed by:

$$\Delta\epsilon_{vs}^p = \frac{1}{h} \cdot \Delta y_\alpha \quad (7)$$



**Figure 2.**  $y_\alpha$ -  $p$  (transformed internal parameter - preconsolidation stress) plane for plastic shakedown.

### 2.2 Elastic behaviour at the equilibrium state

After several wetting and drying cycles, the equilibrium state will be obtained at the end of the suction cycles in the (strain-suction) plane where no plastic strain accumulation can be observed. Consequently, a linear variation of the elastic strain with the suction can be supposed at the equilibrium state. It can be written as:

$$\Delta\epsilon_{vs}^e = \frac{\kappa_s}{v} \cdot \Delta s \quad (8)$$

where,  $\kappa_s$  is the elastic stiffness index for suction variation.

Here, the resilient modulus  $E_r$  is defined and the above equation becomes:

$$\Delta\epsilon_{vs}^e = \frac{1}{E_r} \cdot \Delta s \quad (9)$$

with

$$\frac{1}{E_r} = \frac{\kappa_s}{v} \cdot \frac{1}{s + p_{at}} \quad (10)$$

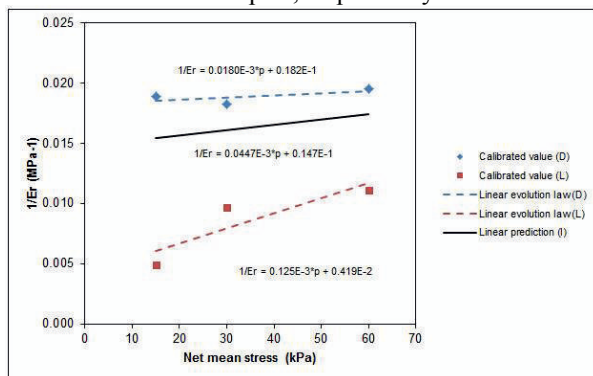
where,  $E_r$  can be calibrated at the elastic equilibrium state.

### 3 Calibration of the parameters for the shakedown-based model

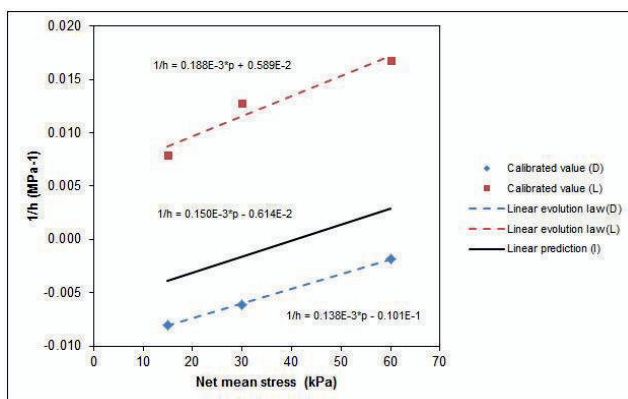
Nowamooz and Masrouri [1,7,20] and Nowamooz et al. [6] studied the hydro-mechanical behaviour of an artificially prepared mixture of 40% silt and 60% bentonite after several drying and wetting cycles. The samples were compacted with the same initial water content ( $w$ ) of 15%. Three different initial dry densities ( $\gamma_d$ ) were 1.27, 1.48 and 1.55  $\text{Mg}\cdot\text{m}^{-3}$ , corresponding to loose (L), intermediate (I) and dense (D) soils, respectively.

Required parameters of the proposed shakedown model are determined by the oedometer tests with suction cycles between 0 and 8 MPa under three constant vertical stresses: 15, 30 and 60 kPa.

Figure 3 and 4 show the evolution law of the inverse of the resilient modulus ( $1/E_r$ ) as well as the inverse of the hardening modulus ( $1/h$ ) with the net mean stress ( $p$ ) for loose and dense samples, respectively.



**Figure 3.** Variation of the inverse of the resilient modulus ( $1/E_r$ ) with net mean stress for the loose and dense samples as well as the prediction for the intermediate samples.

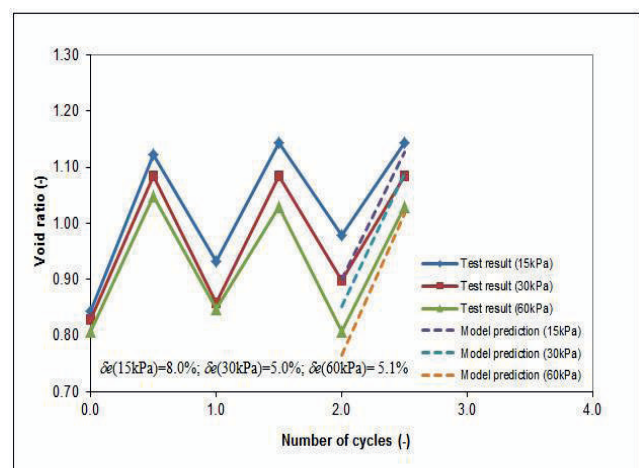


**Figure 4.** Variation of the inverse of the hardening modulus ( $1/h$ ) with net mean stress for the loose and dense samples as well as the prediction for the intermediate samples.

The validation of the model is carried out with the test results obtained for the samples compacted at the

intermediate initial state. The linear variation of the inverse of the elasticity parameter ( $1/E_r$ ) as well as the hardening modulus ( $1/h$ ) with the net mean stresses is interpolated in Figures 3 and 4 for the these samples. For intermediate samples, the threshold value of elastic limit for suction variation ( $s_a$ ) is also considered negligible.

Based on these predicted model parameters, the model validation is conducted for the intermediated samples. Figure 5 represents the comparison between the test results and the model predictions at the different net mean stresses. For these samples, the initial state is closer to the reversible line which need less suction cycles to obtain the equilibrium state. The relative tolerance varies between 5% and 8% confirming the capacity of the proposed model to estimate the accumulated plastic strains.



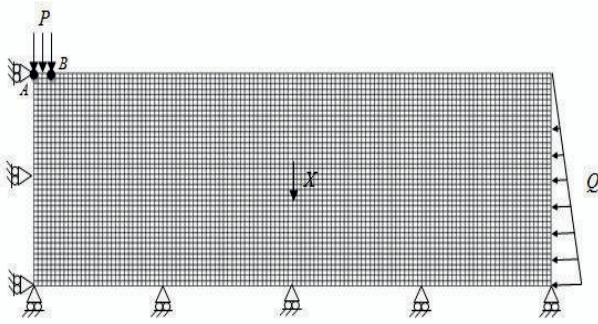
**Figure 5.** Comparisons of test results with model predictions for intermediate sample at the different net mean stresses.

### 4 Numerical simulation of the in-situ behavior of an expansive soil

In this section, the plastic strain field and inelastic displacements of an expansive soil will be simulated after several wetting and drying cycles due to the variation of the climatic conditions, till the stabilized limit state. We use the finite element code CAST3M [21] in which the proposed shakedown-based model has been implemented.

Figure 6 shows the studied geometry 600 cm×200 cm, made up of the intermediately compacted expansive soil (presented in section 3). For the simulations, the geometry is discretized into 4-node quadrilateral elements and 4800 elements have been used to analyze 2-D plane strain problem.

Before shakedown calculation, the in-situ geostatic analysis of the expansive soils is performed where the gravity and lateral stresses are applied to the model (Figure 6). This geostatic stresses are determined by the initial dry density of soil ( $\gamma_d$ ) and the lateral stress coefficient  $K_0$  where these values are summarized in Table 1. Young modulus  $E$  and Poisson ratio  $\nu$  necessary for the elastic mechanical analysis are also given in Table 1.



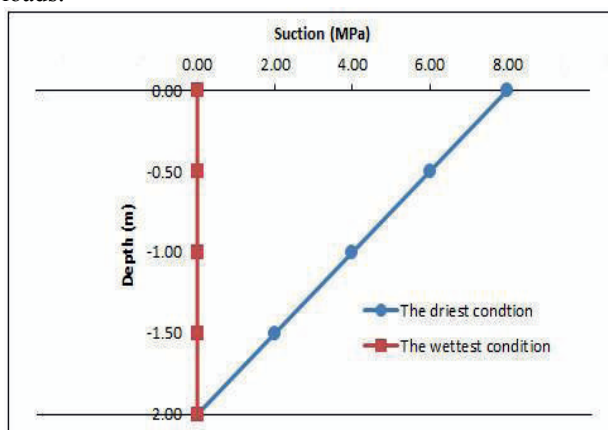
**Figure 6.** Finite element meshing and loading description for 2D geometry simulation.

**Table 1.** Mechanical parameters of the intermediate samples

Parameters	$E(\text{MPa})$	$\nu(-)$	$\gamma_d$	$K_0(-)$
Intermediate soil	18	0.2	1.48	0.5

Different vertical loads ( $P$ ) are applied on the top of the geometry between points A and B in Figure 6 (the length of AB is 30 cm): 15, 30, 60 kPa and the limit stress of 67.85 kPa. The limit stress is 67.85 kPa produces the maximum net mean stress of 60 kPa in the finite element model. Since we obtained the evolution law of shakedown model between 15 and 60 kPa for all the studied materials, the net mean stress values beyond 60 kPa are not taken into account in the numerical modeling.

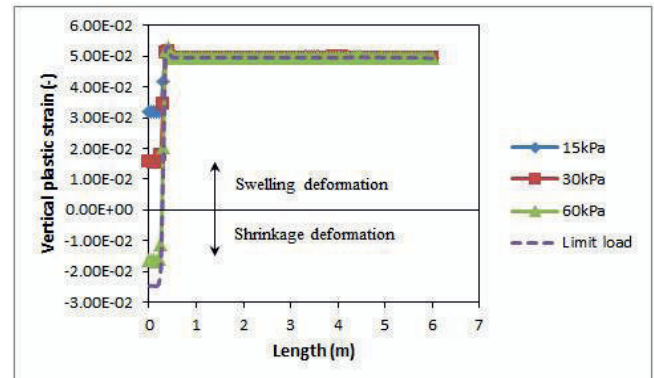
We considered a linear variation of suction with depth as described in Figure 7 in the dry season, while the soil is fully saturated (suction zero) in the wet season. Then, the finite element calculation of the limit state is performed for the wetting and drying cycles between these two extreme conditions under different vertical loads.



**Figure 7.** Suction profile at the driest and the wettest conditions for 2D geometry simulation.

Figure 8 shows the plastic strains on the soil surface ( $y=0$  m) after several wetting and drying cycles for the different vertical stresses. It can be observed that the vertical stress mainly influences the plastic strain field between points A and B. The swelling plastic strains are produced at the vertical stresses of 15 and 30 kPa, while

the accumulated shrinkage deformations are occurred for the higher vertical stresses. Outside AB, the swelling plastic strain reaches its maximum because of null external loads.



**Figure 8.** Finite element calculations of plastic strain field; on the surface ( $y=0$  m) of 2D geometry.

## 5 Conclusion

Several models are developed for the complex hydro-mechanical behavior of the expansive soils submitted to the successive wetting and drying cycles, but these models are based on the traditional incremental method in elasto-plasticity leading to a very large calculation time. To solve this problem, the simplified model based on Zarka shakedown method is developed for expansive soils.

The proposed shakedown model requires three parameters: the linear evolution law of elasticity parameters ( $1/E_r$ ) determined from the equilibrium state at the end of the wetting and drying cycles; the linear evolution law of hardening modulus ( $1/h$ ) calibrated from the accumulated plastic strains during the wetting and drying cycles; and the threshold value of elastic limit for suction variation ( $s_a$ ), taken zero for the sake of simplification.

With the framework of shakedown theory, finite element calculations of a 2D (plane strain) model, made up of intermediate soils was carried out. The soil is subjected to suction cycles between extremely dry and wet conditions under different vertical loads, till the stabilized limit state is achieved. The results show that the swelling plastic strains on the surface of the model ( $y = 0$  m), are produced at the lower vertical stresses while the accumulated shrinkage deformations are occurred for the higher vertical stresses.

Our future development will deal with the activation of the PLC yield surface with the suction cycles to couple our proposed shakedown-based model with the mechanical behavior of the expansive soils.

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